Probability and Statistics: A Primer for Beginners and Pre-Beginners

Prologue to the Prologue: Set Theory

Part One: The Sample Space and Events

Primary reference: Casella-Berger 2^{nd} Edition

In the beginning, there was… Ω

(the sample space)

And in it were…

ALL POSSIBLE OUTCOMES!

(of an experiment)

When a coin was flipped…

$$\Omega = \{ H, T \}$$

When a die was rolled…

$\Omega = \{$  $\}$

There was no limit to what it could contain, even ALL THE NUMBERS!

$$\Omega = (-\infty, \infty)$$

(well, in this case, just the real ones, but you get the idea)

It could be countable:

$$\Omega = \{1, 2, 3, \ldots\}$$

It could be UNcountable:

$$\Omega = (0,\infty) = \{0.1, \ldots$$

0.000001 0.00001 0.0001 0.001 0.01

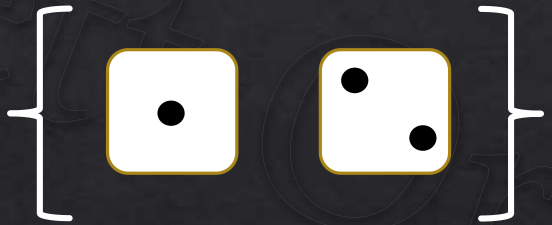
And lo, the elements of Ω were called events.

Like each side of a coin:

Or each face of a die:

Even the combinations of these elements were events.

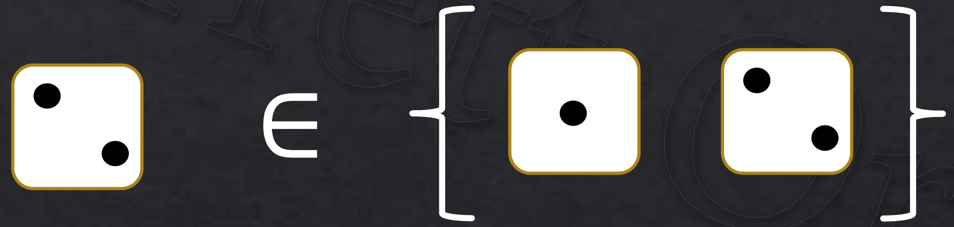
Like rolling a one or two.



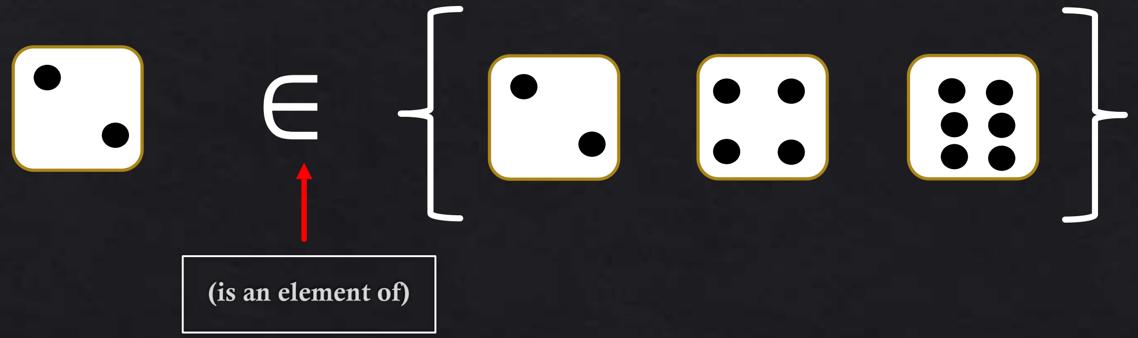
Or an even number

And then the skies darkened, for there was set notation on the horizon… ☹

A single outcome could be an element of a set

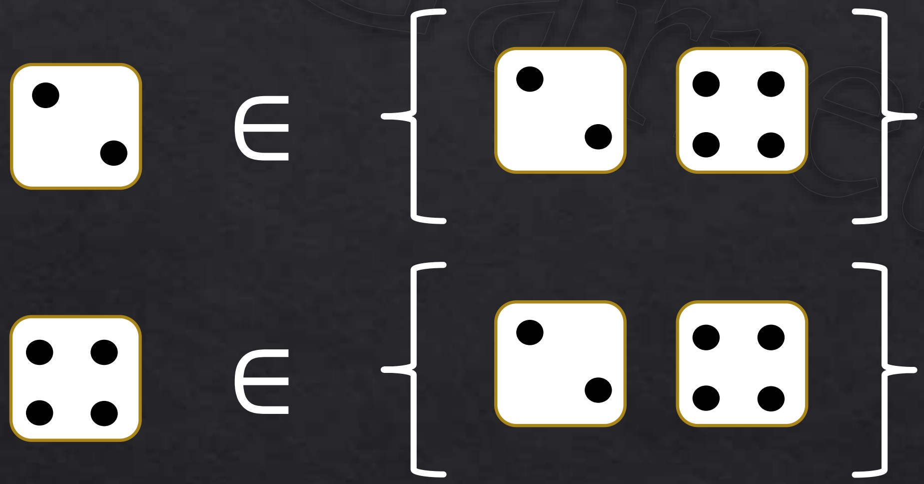


In fact, that outcome was an element of any set that included it

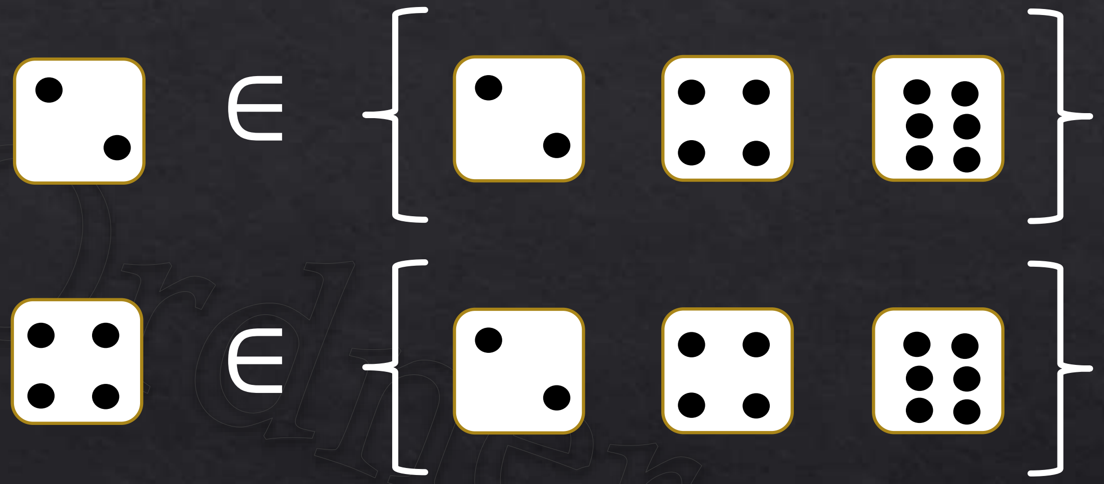


And yet there was more! If every element of one set was also an element of a second set, the first set was considered a subset of the second!

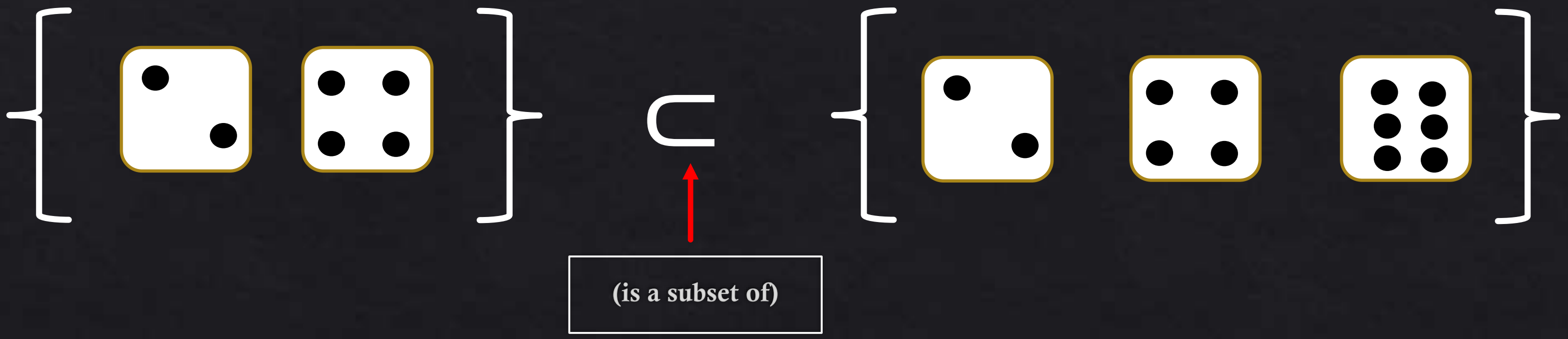
Like this first set,



And this second set,



Meant that…



Then as if to taunt us, all the easy-to-read dice pictures became intimidating algebra!
The two-face became *x*, the four-face became *y*, the six-face became z…

The first set became A…

$$A = \{x, y\}$$

$$x \in A$$

$$y \in A$$

The second set became B…

$$B = \{x, y, z\}$$

$$x \in B$$

$$y \in B$$

And so…

$$A \subset B$$

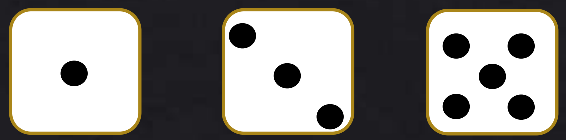
But then they became friendly dice again to explain complementation. If B is the set of even faces, then the odd faces are not in it.

$B = \{$  $\}$

 $\notin B$

(is not an element of)

They comprise its complement, B^C, the set of all elements of Ω that aren't elements of B.

$B^C = \{$  $\}$