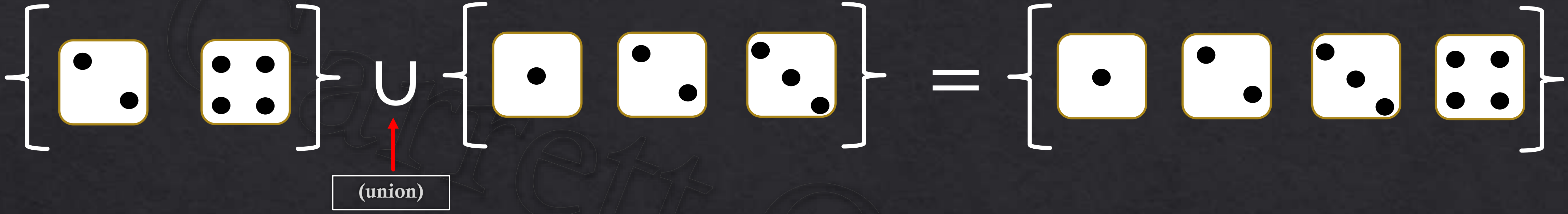
Probability and Statistics: A Primer for Beginners and Pre-Beginners

Prologue to the Prologue: Set Theory

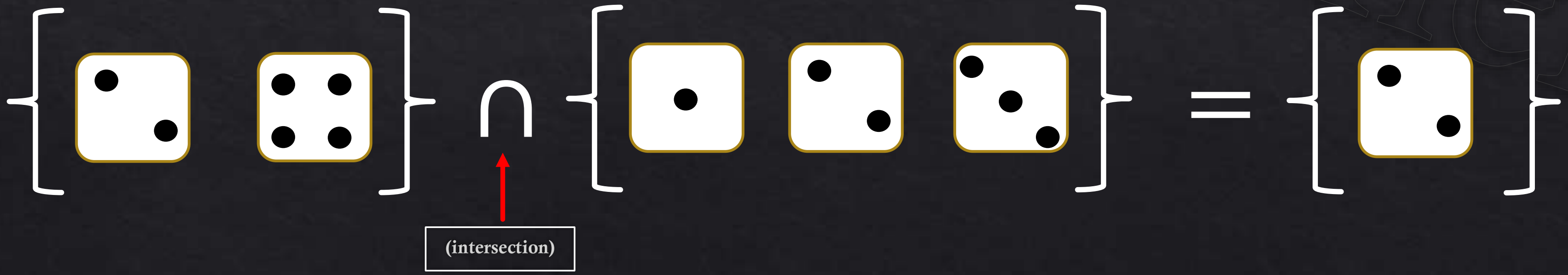
Part Two: Event Operations and Properties

Primary reference: Casella-Berger 2^{nd} Edition

There's other stuff we can do with events. We can combine them, in a union:



We can keep only the outcomes they have in common, in their intersection:



Remember the complementary events from last time?

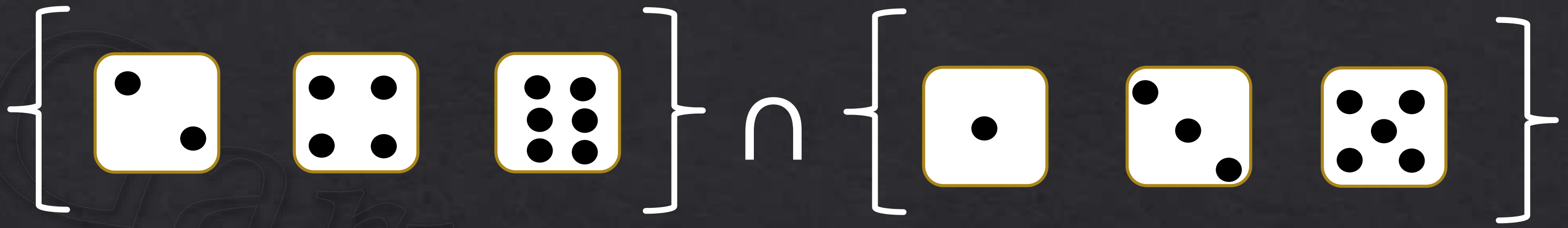
$B = \{$ ⚁ ⚃ ⚅ $\}$, $B^C = \{$ ⚀ ⚂ ⚄ $\}$

Their union has an interesting property:

$\{$ ⚁ ⚃ ⚅ $\} \cup \{$ ⚀ ⚂ ⚄ $\}$

$= \{$ ⚀ ⚁ ⚂ ⚃ ⚄ ⚅ $\} = \Omega$

Their intersection has a far less-interesting property:



$$= \{\ \} = \emptyset$$

(empty set)

Remember we called those events B and B^C? Well, now it doesn't matter what's in them; we can generalize! For any event B and its complement B^C :

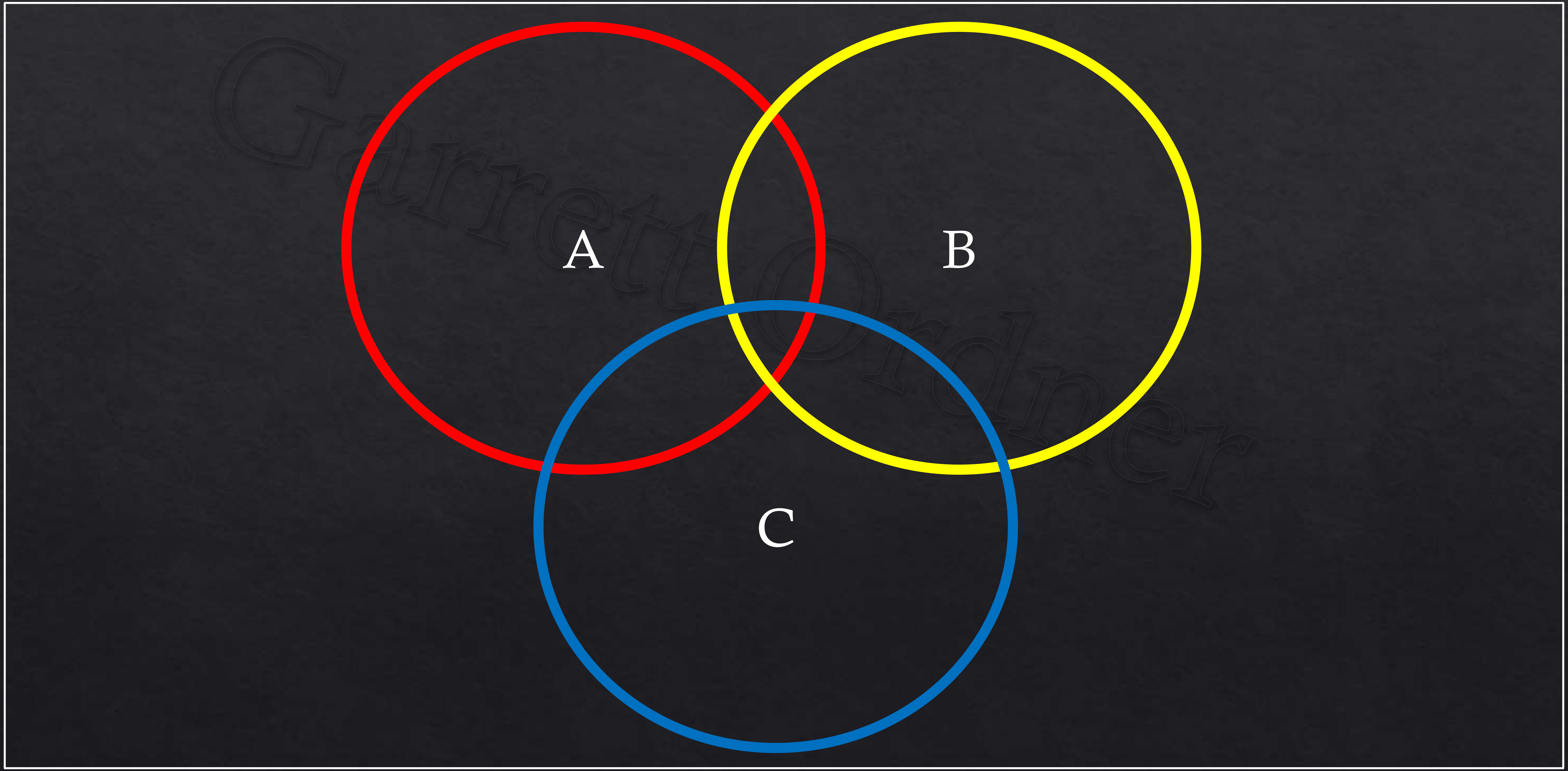
$$B \cup B^C = \Omega$$

$$B \cap B^C = \emptyset$$

Caution!

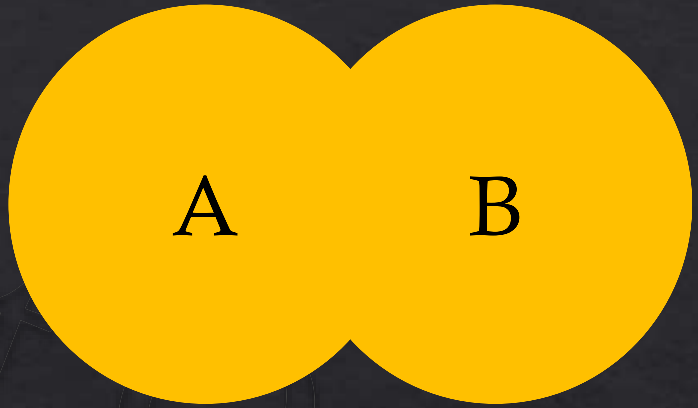
We're about to enter Venn Diagram territory. Casella, Berger, and my old stats theory professor would (presumably) all like me to remind you that diagrams can illustrate, **but they don't prove anything** (and we aren't going to write out many proofs in this course, so go buy the book). Whatever you do, don't draw them on your exams!!!

Let's call this big, ugly rectangle Ω, and put three circular events A, B, and C inside.



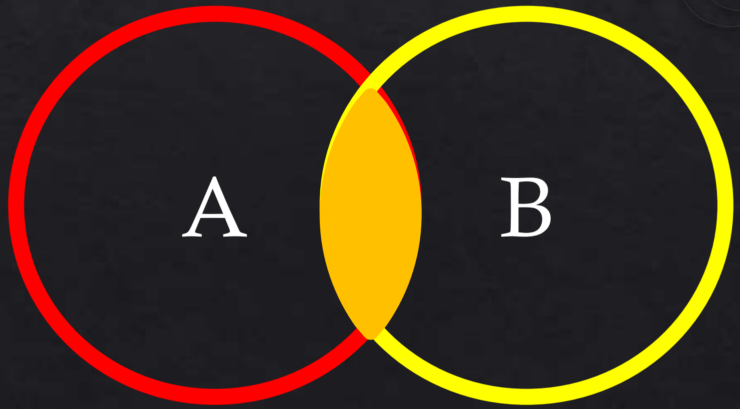
Referring back to that diagram, visualize a union of events as a combination of the circles:

$A \cup B$



And visualize the intersection of the events as the, well, intersection of the circles.

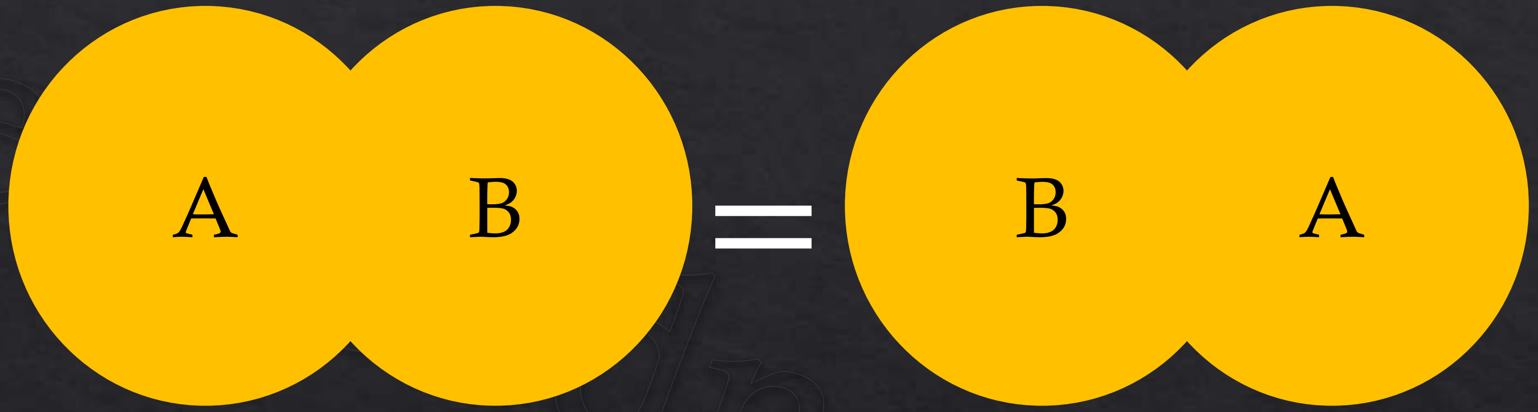
$A \cap B$



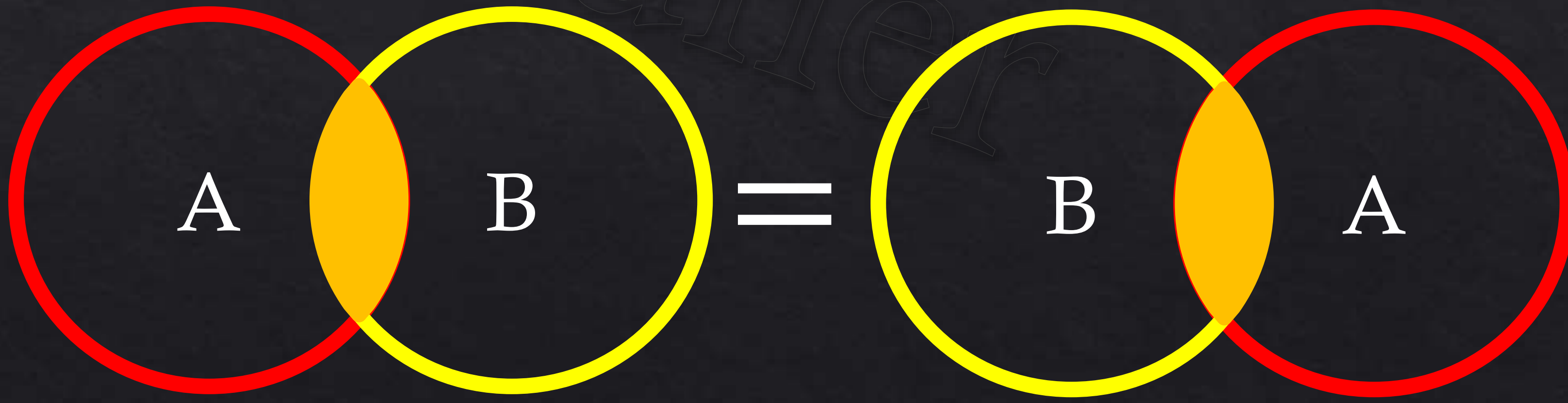
Now we can write and illustrate some simple properties.

Commutative:

$A \cup B = B \cup A$



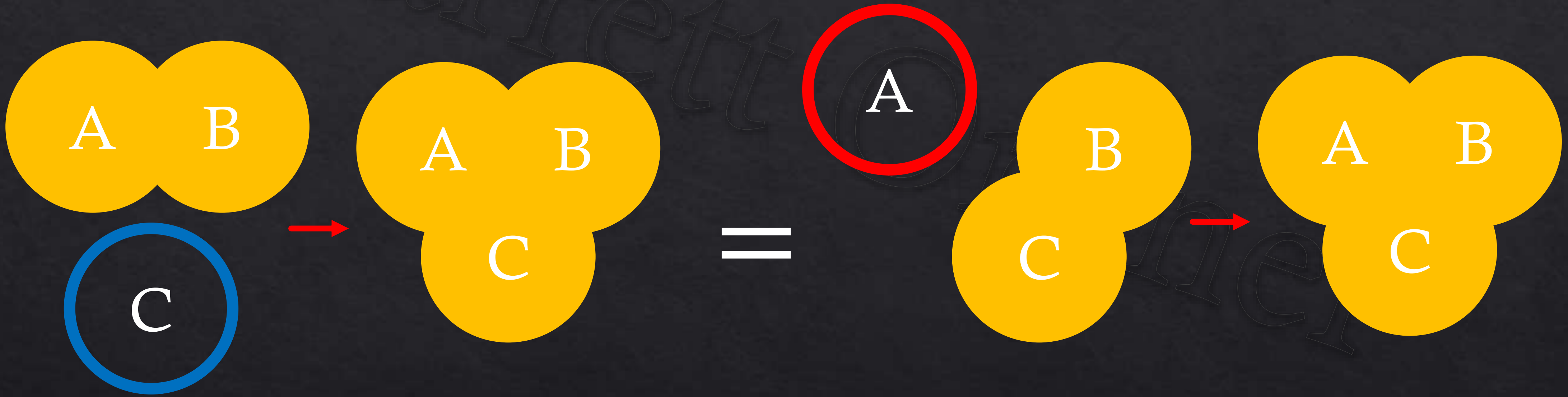
$A \cap B = B \cap A$



Now we can write and illustrate some simple properties.

Associative pt.1:

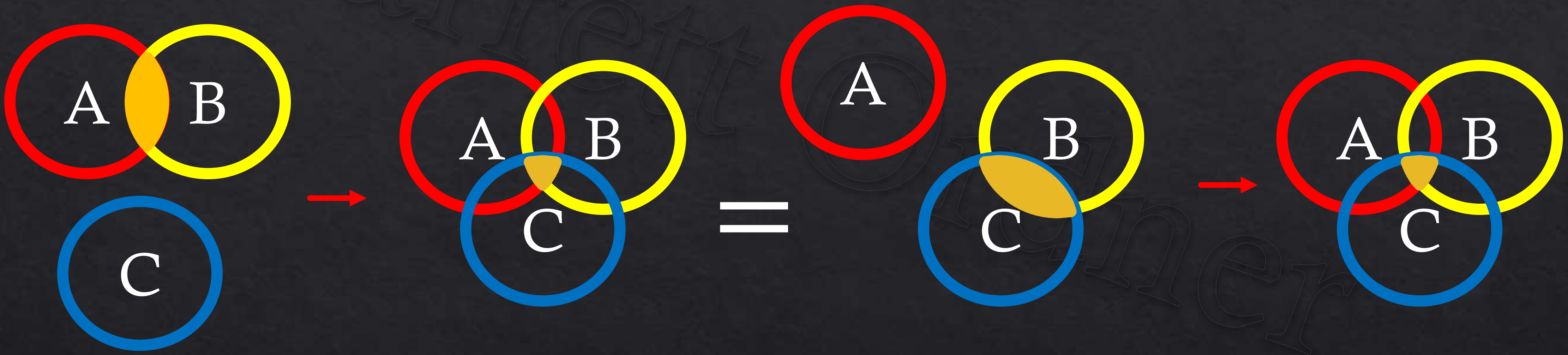
$$(A \cup B) \cup C = A \cup (B \cup C)$$



Now we can write and illustrate some simple properties.

Associative pt.2:

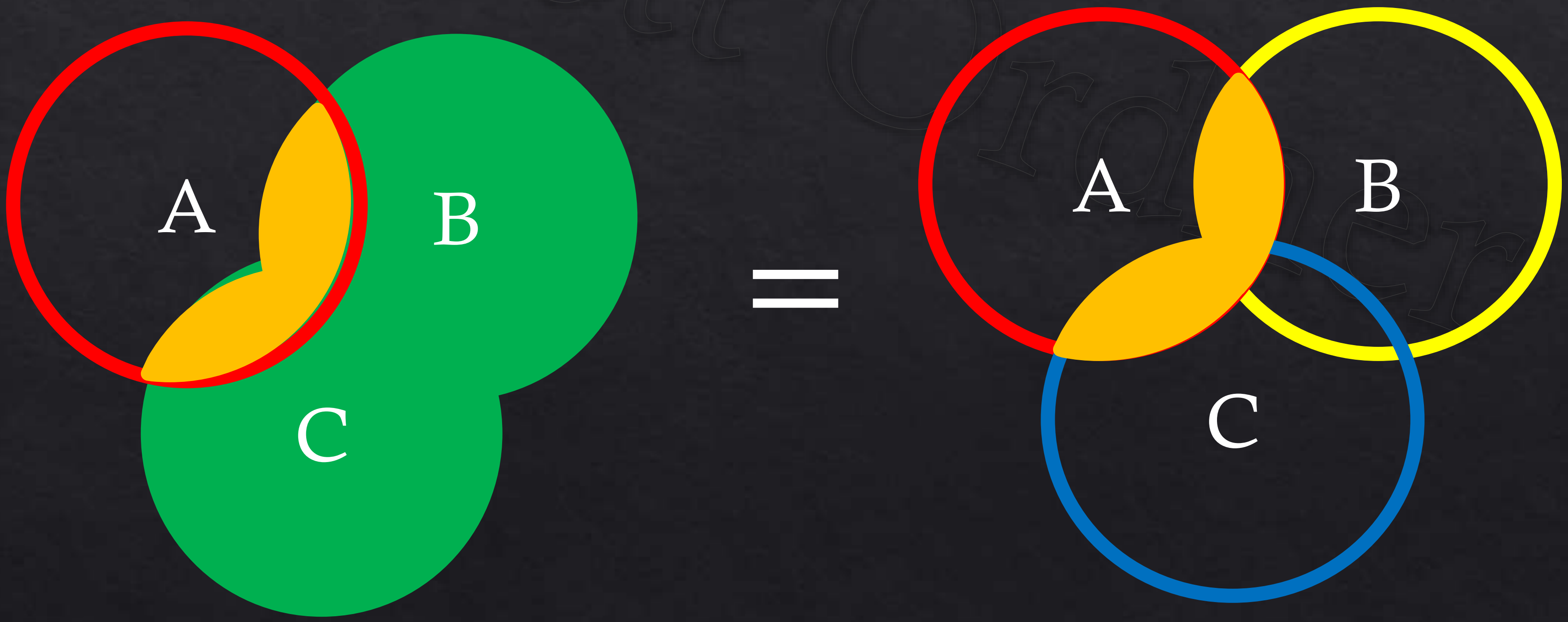
$$(A \cap B) \cap C = A \cap (B \cap C)$$



Now we can write and illustrate some simple properties.

Distributive, pt.1:

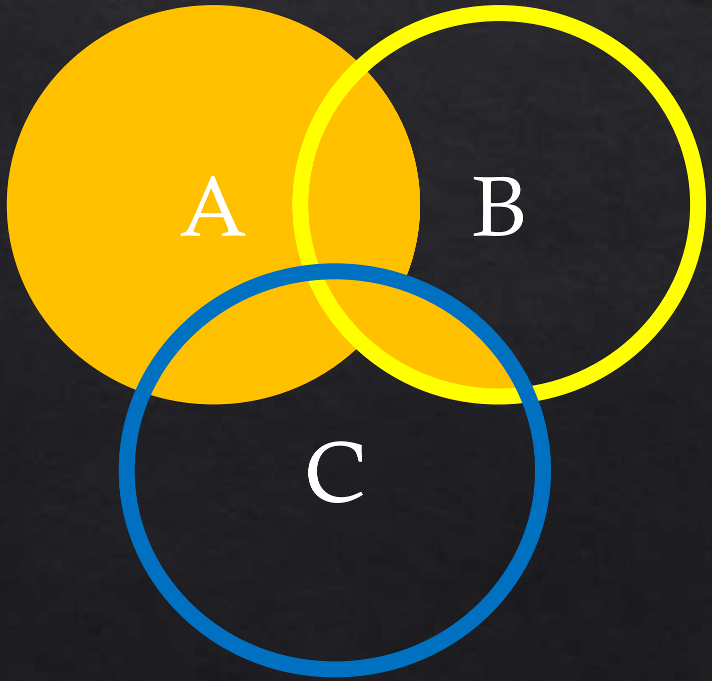
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



Now we can write and illustrate some simple properties.

Distributive, pt.2:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$



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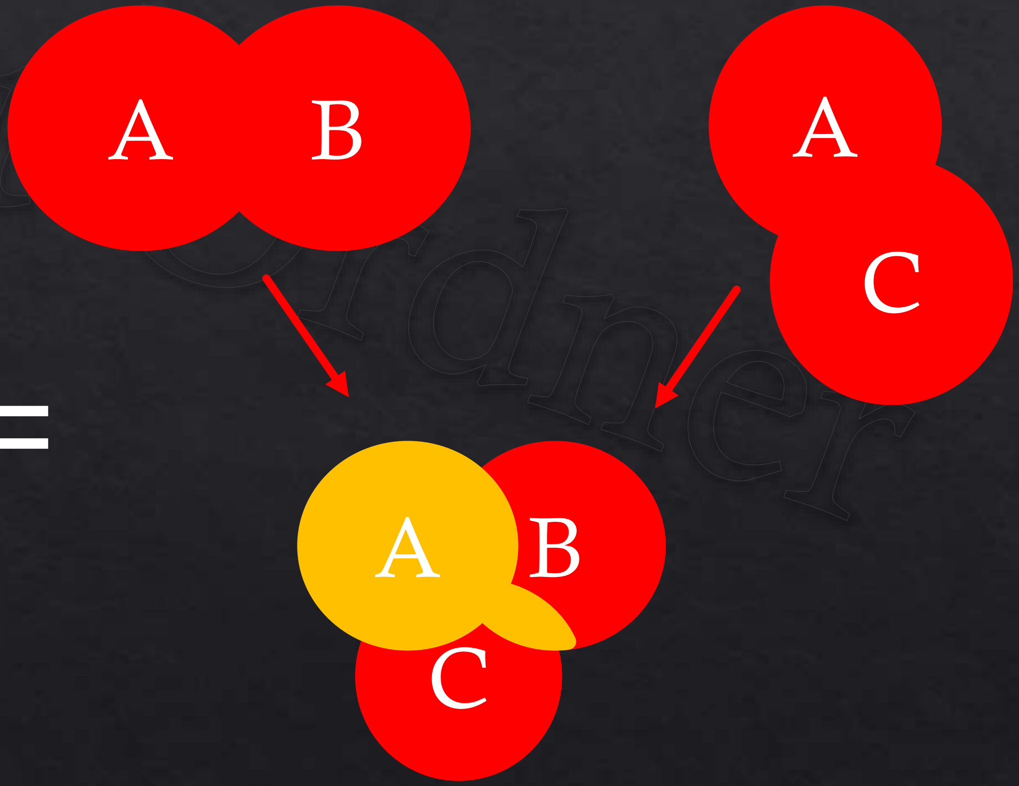
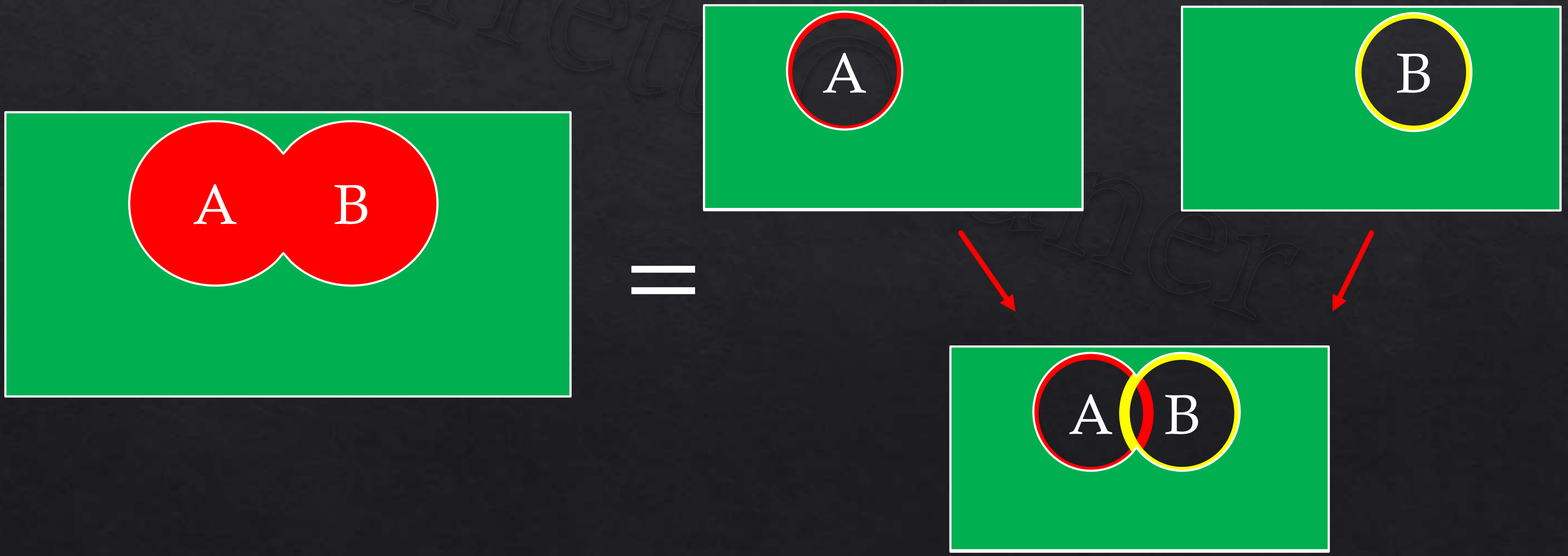


Now we can write and illustrate some simple properties.

DeMorgan's Laws, pt. 1:

$$(A \cup B)^C = A^C \cap B^C$$



Now we can write and illustrate some simple properties.

DeMorgan's Laws, pt. 2:

$$(A \cap B)^C = A^C \cup B^C$$