Probability and Statistics: A Primer for Beginners and Pre-Beginners

The Journey Begins: Probability Theory

Part Five: Learning to Count (pt.1)

Primary reference: Casella-Berger 2nd Edition

…but I know how to count.

Do you, though? Let's try something, then. See those nine balls below? Say you need to choose four of them for a lottery, and the ticket has to get all four numbers right *in the same order they were picked* to win.

1 2 3 4 5 6 7 8 9

How many different lottery tickets can there be?

Hmm…

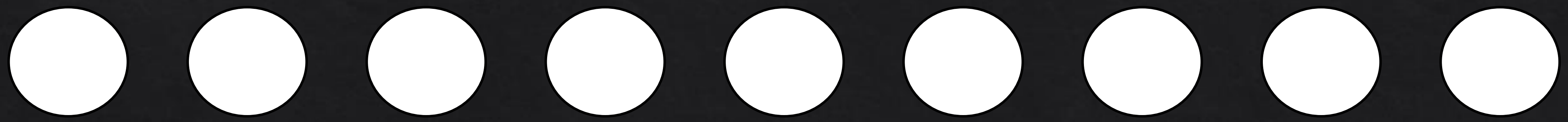
- 1-2-3-4
- 1-2-3-5
- 1-2-3-6
- 1-2-3-7
- 1-2-3-8
- 1-2-3-9
- 1-2-4-3
- 1-2-4-5
- 1-2-4-6
- 1-2-4-7
- 1-2-4-8
- 1-2-4-9
- …

All right, stop! This'll take forever.

It sure will! And just imagine if I'd used the 44-number lottery from Casella and Berger's example! But we can think about this a little differently.

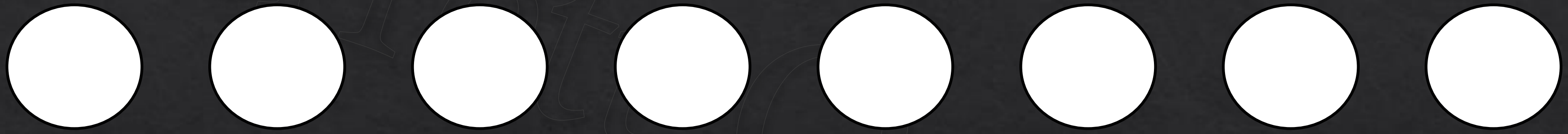
We're choosing from these nine balls *without replacing them*, right?

So for our first pick, there are nine possibilities:

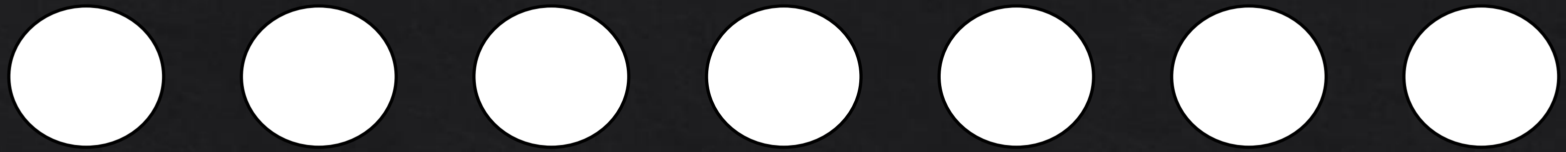


Yeah, ok.

Then, no matter what that number was, there are 8 balls left.

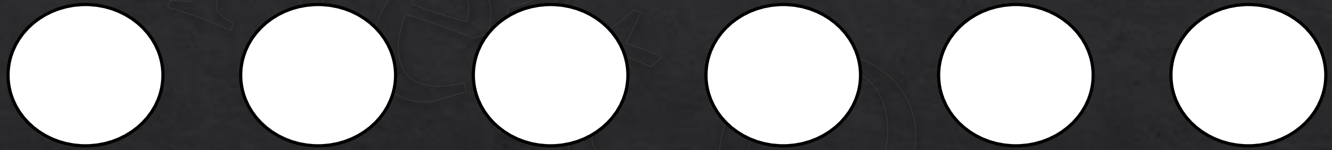


So, for each of the 9 possible first choices, there are 8 possible *second* choices. So there are 9*8 possible *permutations* for the first two numbers. And after the second pick, there are 7 balls left:



…so, 9*8*7 possible permutations for the first three choices?

Now you're getting it! And that leaves 6 possible final choices.



So how many possible lottery tickets? 9*8*7*6 = 3,024

Choosing that lottery ticket consisted of four tasks (each number to pick). The first task could be done 9 ways, the second could be done 8 ways, the third could be done 7 ways, and the fourth could be done 6 ways…

I sense a theorem coming on…

How perceptive of you!

If a job consists of k tasks, and the ith task can be done n_i ways, then the total number of ways to complete the task is

$$n_1 * n_2 * n_3 * \cdots * n_k$$

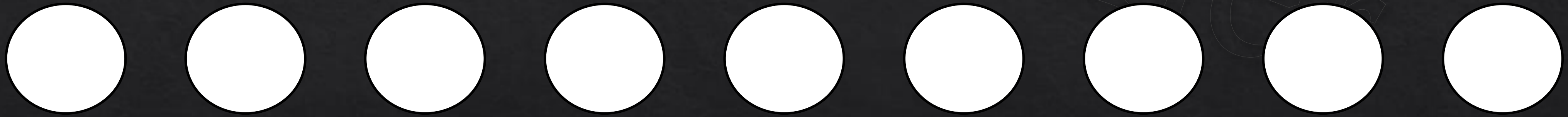
Level up!

You just learned the Fundamental Theorem of Counting!

Neat. So we’re done, then?

No! ☺

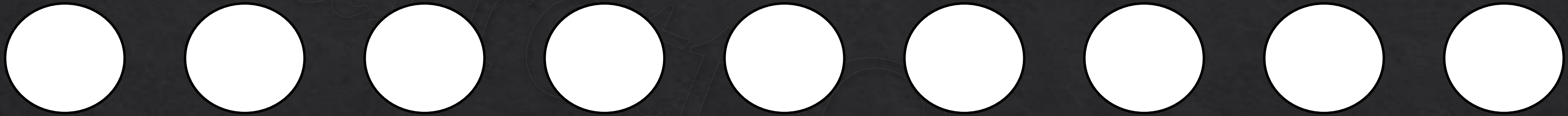
Say that first lottery was hemorrhaging money, so the organizers changed the rules: Now, after being picked, the ball gets *put back* and could be picked again! We still have nine choices for the first number:



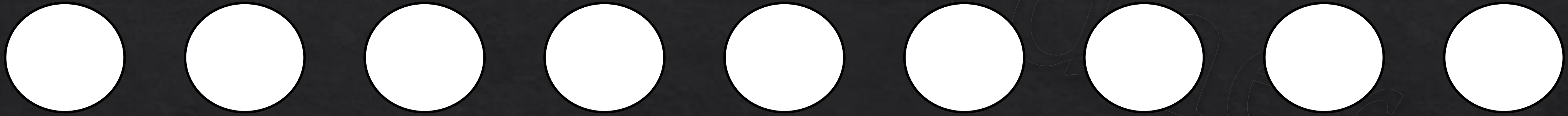
But then, things change…

Right, we've still got nine choices for the second number.

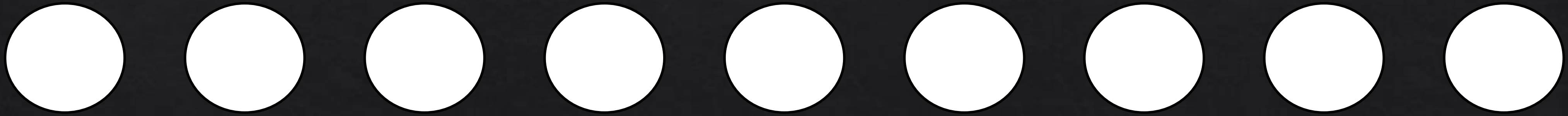
Exactly:



Same goes for the third number:



And the fourth:



Can we still use the theorem?

We sure can!

Our "job" still consists of 4 tasks, but now *each* of those tasks can be done in nine ways, giving us

$$n_1 * n_2 * n_3 * n_4 = 9 * 9 * 9 * 9 = 9^4 = 6{,}561 \text{ possible tickets}$$